



Supersymmetry breaking by higher dimension operators [☆]

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Abstract

We discuss a supersymmetry breaking mechanism for $\mathcal{N} = 1$ theories triggered by higher dimensional operators. We consider such operators for real linear and chiral spinor superfields that break supersymmetry and reduce to the Volkov–Akulov action. We also consider supersymmetry breaking induced by a higher dimensional operator of a nonminimal scalar (complex linear) multiplet. The latter differs from the standard chiral multiplet in its auxiliary sector, which contains, in addition to the complex scalar auxiliary of a chiral superfield, a complex vector and two spinors auxiliaries. By adding an appropriate higher dimension operator, the scalar auxiliary may acquire a nonzero vev triggering spontaneous supersymmetry breaking. We find that the spectrum of the theory in the supersymmetry breaking vacuum consists of a free chiral multiplet and a constraint chiral superfield describing the goldstino. Interestingly, the latter turns out to be one of the auxiliary fermions, which becomes dynamical in the supersymmetry breaking vacuum. In all cases we are considering here, there is no sgoldstino mode and thus the goldstino does not have a superpartner. The sgoldstino is decoupled since the goldstino is one of the auxiliaries, which is propagating

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only in the supersymmetry breaking vacuum. We also point out how higher dimension operators introduce a potential for the propagating scalar of the theory.

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1. Introduction

Supersymmetry is one of the most appealing candidates for new physics. It has not been observed so far; thus, it should be broken at some high energy scale if it is realized at all. The central role on how supersymmetry is broken is usually played by the scalar potential of the supersymmetry breaking sector. Scalar potentials in supersymmetry and supergravity have extensively been studied for two-derivative theories. Even though it is known that introducing higher dimension operators spoils the form of the scalar potential, it seems that the theory somehow protects itself from unconventional non-supersymmetric vacua [1]. Our task here is to discuss how scalar potentials are modified and may lead to supersymmetry breaking when higher dimension operators are introduced. The Goldstone fermion associated with the supersymmetry breaking, the goldstino, is described by the Volkov–Akulov action [2], in which supersymmetry is non-linearly realized. In particular, the goldstino dynamics has been related in [3] to the superconformal anomaly multiplet X corresponding to the FZ supercurrent [4]. The multiplet of anomalies X , defined in the UV flows in the IR, under renormalization group, to a chiral superfield X_{NL} which obeys the constraint $X_{NL}^2 = 0$. This constrained superfield is the realization of the goldstino given in [5]. Since the dynamics of the goldstino is universal, the IR action in [3] is the same as in [5]. Constrained superfields have been used before to accommodate the goldstino. Indeed, there are alternative formulations in which the goldstino sits in a constrained superfield, such as a constrained chiral multiplet [6], a constrained vector multiplet [7], a spinor superfield [8], or a complex linear superfield [9]. Constrained superfields have also been used recently in the MSSM context [10–13] and in inflationary cosmology, where the inflaton is identified with the sgoldstino [14]. In addition their interaction with matter has been worked out in [15].

Supersymmetric theories that contains higher dimension operators (derivative or non-derivative ones) have some novel features [16–19]. Among these, an interesting aspect is that higher dimension operators can contribute to the scalar potential. This has been discussed earlier in [1] where a few examples have been given. In particular, theories with no potential at the leading two-derivative level, may develop a nontrivial potential when higher dimension operators are taken into account and may even lead to supersymmetry breaking, as already mentioned above. At this point there are however, two dangerous aspects. The first one concerns the appearance of ghost instabilities. In the type of theories we are discussing, this instability is not present as the theory does not have those higher derivatives terms which might give rise to such dangerous states. The second issue concerns the auxiliary fields. Here, we are still able to eliminate the auxiliaries of the multiplet since they appeared algebraically in the supersymmetric Lagrangian.

We will consider various theories exhibiting supersymmetry breaking in the presence of higher dimension operators. Special attention will be devoted to a globally supersymmetric model for a complex linear multiplet. As we will explain in one of the following sections, the complex linear multiplet, or nonminimal multiplet, contains the degrees of freedom of a chiral multiplet and in addition, two fermions and a complex vector. At the two derivative level, both the extra fermions and the complex vector are auxiliaries and can be integrated out, giving on-shell just a free complex scalar and a fermion. Due to the constraints the complex linear satisfies, there is no superpotential one can write down and the introduction of an F-term for

non-derivative interactions is not possible. So, one relies on modifying the D-term in order to get some non-trivial interactions and an emerging potential induced by higher dimension operators [1,17–19]. Under certain conditions, it may happen that the new potential develops another extremum for the auxiliaries which break supersymmetry. In this case, new phases will emerge, only one of which will be realized when the higher dimension operators interactions are turned off. It should be noted however, that these new phases are not different phases of the same theory, but rather different theories. The examples studied in [1] were not successful in this respect, basically because the auxiliaries appeared in the higher derivative terms with the same sign as in the leading two-derivative term. This has the effect that the minimum of the potential is stable with respect to the addition of the higher dimension term. However, in the case of the complex linear multiplet, the auxiliary in the two derivative term and in the higher derivative term appear with opposite sign. This has the effect of introducing now a new minimum for a non-zero value of the auxiliary, thereby breaking supersymmetry. The interesting phenomenon that appears here is that the goldstino turns out to be one of the auxiliary fermions of the multiplet, which in the new vacuum acquires a kinetic term, but vanishes in the supersymmetric vacuum of the theory. After integrating out the auxiliaries, we are left with a complex scalar, a fermion and a goldstino without supersymmetric partner, as supersymmetry is broken. Therefore, there is a mismatch of bosonic and fermion degrees of freedom as for example in Volkov–Akulov type of models where supersymmetry is non-linearly realized [2].

This paper is organized as follows. In the next section we present theories with higher dimensional operators that exhibit susy breaking and the corresponding Volkov–Akulov actions. In Section 3 we describe the complex linear multiplet. In Section 4 we show how higher dimensional operators of the complex linear multiplet may lead to susy breaking and we prove the equivalence to non-linear realizations. Finally, we conclude in the last Section 5.

2. SUSY breaking and Volkov–Akulov actions

One of the explicit examples considered in [1] to demonstrate that the scalar potential is sensitive to the addition of higher dimension terms, is a supersymmetric σ -model with four-derivative coupling. Its standard Lagrangian is²

$$\mathcal{L}_\sigma = \int d^4\theta K(\Phi, \bar{\Phi}), \quad (1)$$

where $K(\Phi, \bar{\Phi})$ is the Kähler potential. The latter can be considered as a composite vector multiplet possessing an effective gauge (Kähler) invariance

$$K \rightarrow K + i(S - \bar{S}), \quad (2)$$

where S is a chiral superfield. As we are going to keep this invariance for the higher dimension operators as well, we will construct the latter in terms of the superfield field strength

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha K \quad (3)$$

for the composite vector $K(\Phi, \bar{\Phi})$. Then, clearly, the most general Kähler invariant Lagrangian up to four-derivative terms is

² Our superspace conventions can be found in [20].

$$\mathcal{L}_\sigma = \int d^4\theta K(\Phi, \bar{\Phi}) + \left(\int d^2\theta g(\Phi) + \lambda \int d^2\theta W^2(K) + \text{h.c.} \right) \quad (4)$$

where $g(\Phi)$ is the superpotential and $\lambda > 0$. Without loss of generality, let us consider the simplest case of a single chiral multiplet with $K = \Phi \bar{\Phi}$ and $g(\Phi) = 0$. Then Eq. (4) turns out to be

$$\mathcal{L}_\sigma = \int d^4\theta \left(\Phi \bar{\Phi} + \frac{\lambda}{2} D^\alpha \Phi D_\alpha \Phi \bar{D}_{\dot{\alpha}} \bar{\Phi} \bar{D}^{\dot{\alpha}} \bar{\Phi} \right) \quad (5)$$

and the scalar potential turns out to be [1]

$$-V_F = |F|^2 + 8\lambda |F|^4. \quad (6)$$

The minimum of the potential is at $F = 0$, which is also the minimum of the theory in the $\lambda \rightarrow 0$ limit.

2.1. Chiral spinor superfield

There are other possibilities one may wish to consider. For example, let us consider the Lagrangian (cf. [3,5])

$$\mathcal{L}_W = \frac{1}{4} \left(\int d^2\theta W^\alpha W_\alpha + \text{h.c.} \right) + \frac{1}{\Lambda^4} \int d^4\theta W^\alpha W_\alpha \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}, \quad (7)$$

where

$$W_\alpha = \lambda_\alpha + \theta_\alpha D + \theta^\beta F_{\alpha\beta} + \theta^2 \chi_\alpha, \quad (8)$$

so that W_α is chiral but otherwise unconstrained and $F_{\alpha\beta} = F_{\beta\alpha}$.

The component form of the Lagrangian (7) is

$$\begin{aligned} \mathcal{L}_W = & \frac{1}{4} \left(D^2 + 2\chi\lambda + \frac{1}{2} F^{\alpha\beta} F_{\alpha\beta} + \text{h.c.} \right) \\ & + \frac{1}{\Lambda^4} \left[\lambda^2 \partial^2 \bar{\lambda}^2 + \left(D^2 + 2\chi\lambda + \frac{1}{2} F^2 \right) \left(\bar{D}^2 + 2\bar{\chi}\bar{\lambda} + \frac{1}{2} \bar{F}^2 \right) \right] \\ & - i \frac{1}{\Lambda^4} (\lambda^\alpha D - F^{\alpha\beta} \lambda_\beta) \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu (\bar{\lambda}^{\dot{\alpha}} \bar{D} - \bar{F}^{\alpha\beta} \bar{\lambda}_{\dot{\beta}}) \end{aligned} \quad (9)$$

where

$$F^{\alpha\beta} = \epsilon^{\alpha\sigma} \epsilon^{\beta\rho} F_{\sigma\rho}. \quad (10)$$

In the particular case that W_α is the field-strength superfield and satisfies $D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$, the Lagrangian has been worked out in [1,19]. The Lagrangian (7) is of the form [3,5]

$$\mathcal{L}_W = \int d^4\theta X \bar{X} + \frac{\Lambda^4}{4} \left(\int d^2\theta X + \text{h.c.} \right) \quad (11)$$

where $X = W^\alpha W_\alpha$ satisfies

$$X^2 = 0. \quad (12)$$

The explicit form of X is

$$X = W^\alpha W_\alpha = \lambda^2 + 2\theta^\beta (\epsilon_{\beta\alpha} D - F_{\beta\alpha}) \lambda^\alpha + \left(\frac{1}{2} F^{\alpha\beta} F_{\alpha\beta} + D^2 + 2\chi\lambda \right) \theta^2 \quad (13)$$

with $F^{\alpha\beta} = \epsilon^{\alpha\rho} \epsilon^{\beta\sigma} F_{\rho\sigma}$. By defining

$$G_\beta = 2\lambda_\beta D - 2F_{\beta\alpha} \lambda^\alpha \quad (14)$$

and noticing that, because $\lambda^2 \lambda_\alpha = 0$,

$$G^2 = \lambda^2 (4D^2 + 2F^{\alpha\beta} F_{\alpha\beta}) = \lambda^2 (4D^2 + 2F^{\alpha\beta} F_{\alpha\beta} + 8\chi\lambda) \equiv 4\lambda^2 \mathcal{F}, \quad (15)$$

we get the parametrization of X in chiral coordinates [3,5]

$$X = \frac{\tilde{G}^2}{2\mathcal{F}} + \sqrt{2}\theta\tilde{G} + \theta^2\mathcal{F}. \quad (16)$$

Here we have rescaled $G = \sqrt{2}\tilde{G}$. In a sense, W_α is the square root of the goldstino. If the above form of X is plugged back in Eq. (11), the Volkov–Akulov Lagrangian for the goldstino G is obtained [3,5].

We should note here that the resulting Lagrangian is written entirely in terms of the goldstino G_α . One would expect the theory to propagate also its supersymmetric partner, the sgoldstino to fill together a multiplet of the (broken) susy. However, it seems that the sgoldstino has been integrated out from the theory. This is due to the fact that the original multiplet didn't have any propagating fields as both fermions χ, λ and bosons $D, F_{\alpha\beta}$ were auxiliaries. In a sense, the original theory can be considered as the zero-momentum limit (or infinite mass limit) of a theory where all fields were propagating. This is equivalent to sgoldstino decoupling [3,5,10,13,15] and we correctly find here that the goldstino is the only propagating mode in the susy broken branch.

A way to find the vev of \mathcal{F} is from the bosonic part of (7), which turns out to be

$$\mathcal{L}_W^B = \left(\frac{1}{8} F^{\alpha\beta} F_{\alpha\beta} + \frac{1}{4} D^2 + \text{h.c.} \right) + \frac{1}{\Lambda^4} \left(D^2 + \frac{1}{2} F^{\alpha\beta} F_{\alpha\beta} \right) \left(\bar{D}^2 + \frac{1}{2} \bar{F}^{\dot{\alpha}\dot{\beta}} \bar{F}_{\dot{\alpha}\dot{\beta}} \right). \quad (17)$$

There are now two solutions for D ,

$$(i) \quad D = 0, \quad (18)$$

$$(ii) \quad D^2 = -\frac{1}{2} F^{\alpha\beta} F_{\alpha\beta} - \frac{\Lambda^4}{4}, \quad \bar{D}^2 = -\frac{1}{2} \bar{F}^{\dot{\alpha}\dot{\beta}} \bar{F}_{\dot{\alpha}\dot{\beta}} - \frac{\Lambda^4}{4}. \quad (19)$$

The first solution is the supersymmetric Lorentz-invariant vacuum, provided $F_{\alpha\beta} = 0$, whereas the second solution gives

$$\mathcal{F} = -\frac{\Lambda^4}{4}. \quad (20)$$

Then $\langle F_{\alpha\beta} \rangle \neq 0$ clearly breaks supersymmetry but also Lorentz invariance at the same time. However, it is possible to preserve Lorentz invariance if $\langle F_{\alpha\beta} \rangle = 0$ and $\langle F^{\alpha\beta} F_{\alpha\beta} \rangle \neq 0$ as required by (19).

In the particular case in which W_α is the field strength superfield, the bosonic part of (7) turns out to be [19]

$$\begin{aligned} \mathcal{L}_W^B = & -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{i}{8} \epsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} F_{\kappa\lambda} + \frac{1}{2} D^2 \\ & + \frac{1}{\Lambda^4} \left\{ \frac{1}{4} (F^{\mu\nu} F_{\mu\nu})^2 - F^{\mu\nu} F_{\mu\nu} D^2 + \frac{1}{16} (\epsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} F_{\kappa\lambda})^2 + D^4 \right\}. \end{aligned} \quad (21)$$

There are two solutions for D ,

$$(i) \quad D = 0, \quad (22)$$

$$(ii) \quad D^2 = \frac{1}{2} F^{\mu\nu} F_{\mu\nu} - \frac{\Lambda^4}{4}. \quad (23)$$

The first solution corresponds to the supersymmetric branch, whereas the second solution gives the possibility $\langle D^2 \rangle \neq 0$ and may break supersymmetry. However, this is not a Lorentz-invariant vacuum, since (23) requires a non-vanishing $F^{\mu\nu} F_{\mu\nu}$ for supersymmetry breaking. In particular, since D^2 is positive, this vacuum can only be sustained with a non-zero background magnetic field.

2.2. Real linear multiplet

Another interesting example is provided by the Lagrangian

$$L = \int d^4\theta \left(-L^2 + \frac{1}{64\Lambda^4} D^\alpha L D_\alpha L \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L \right), \quad (24)$$

where L is a real linear multiplet. The Grassmann expansion of the latter may be written as

$$L = \phi + \theta\psi + \bar{\theta}\bar{\psi} - \theta\sigma_\mu\bar{\theta}H^\mu - \frac{i}{2}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi + \frac{i}{2}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\psi} - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi \quad (25)$$

and satisfies

$$L = \bar{L}, \quad D^2 L = 0. \quad (26)$$

This implies that the vector H_μ is divergenceless

$$\partial^\mu H_\mu = 0. \quad (27)$$

The action (24) can be written as

$$L = \int d^4\theta \left(-L^2 + \frac{1}{64\Lambda^4} X \bar{X} \right) = \int d^4\theta \left(\frac{1}{64\Lambda^4} X \bar{X} \right) + \left(\frac{1}{4} \int d^2\theta X + \text{h.c.} \right), \quad (28)$$

with

$$\bar{X} \equiv D^\alpha L D_\alpha L = \frac{1}{2} D^2 L^2. \quad (29)$$

Note that \bar{X} is antichiral, so X is chiral and obeys $X^2 = 0$. Then the Lagrangian (28) is the same as in [3,5] (modulo normalization factors). In particular, X is explicitly written in chiral coordinates as

$$X = \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = \bar{\psi}^2 - 2\theta\sigma_\mu\bar{\psi}(i\partial^\mu\phi + H^\mu) + \theta^2[2i\partial^\mu\psi\sigma_\mu\bar{\psi} + (i\partial^\mu\phi + H^\mu)^2] \quad (30)$$

therefore, it is chiral with auxiliary field \mathcal{F}

$$\mathcal{F} = (i\partial_\mu\phi + H_\mu)(i\partial^\mu\phi + H^\mu). \quad (31)$$

The goldstino now is given by

$$G_\alpha = -2\sigma_{\mu\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}(i\partial^\mu\phi + H^\mu). \quad (32)$$

It is easy to see that the bosonic part of (28) is

$$\mathcal{L}^B = \frac{1}{2} H_\mu H^\mu - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{64\Lambda^4} |(i\partial_\mu \phi + H_\mu)^2|^2. \quad (33)$$

There is a supersymmetric vacuum $H_\mu = 0$, $\phi = \text{const.}$ and a supersymmetry breaking one (with $\phi = \text{const.}$)

$$H_\mu H^\mu = -16\Lambda^4. \quad (34)$$

In this case, supersymmetry is broken and the theory reduces to the standard Volkov–Akulov for the goldstino G . In spite of appearances, the vacuum solution (34) does not break Lorentz invariance, since the divergenceless vector H_μ and $\partial_\mu \phi$ combine into the unconstrained vector A_μ , which does not propagate, because it has algebraic equations of motion. Therefore, a nonzero constant vev for A_μ does not affect the dynamics since it either disappears from the Lagrangian or it arranges itself into Lorentz-invariant composite quantities. We also note that, after using (30), the action (28) is written entirely in terms of the goldstino field G_α . Again here, similarly to the spinor superfield case above, there is no superpartner of the goldstino. The sgoldstino is decoupled as all fields before susy breaking were auxiliaries and therefore (28) may be considered as the zero-momentum limit of a theory where these were propagating. In this limit, the sgoldstino decouples and the theory describes a Volkov–Akulov model.

3. Validity of the Volkov–Akulov description

The theories above, as well as the one we will examine later, must be understood as effective IR theories. If a supersymmetric UV completion existed, then the sgoldstino φ would have a large but finite mass m_s . It would interact with the goldstino through terms of the schematic form

$$\kappa G_\alpha G^\alpha \varphi + (m_s^2/2)\varphi^2 + \dots, \quad (35)$$

with a coupling constant $\kappa = O(m_s^2/f)$. At energies below m_s , the sgoldstino fields can be integrated out, producing additional irrelevant operators weighted by inverse powers of the new scale $\Lambda' = f/m_s$. Curiously, these additional interactions become negligible when the sgoldstino is massive but *lighter* than \sqrt{f} : $\Lambda' \gg \sqrt{f} \rightarrow m_s \ll \sqrt{f}$. We will explicitly demonstrate this in the case of supersymmetric theories with chiral multiplets. Note that here f is the vev of the auxiliary F , and in order to avoid any confusion of the scale Λ with the cutoff, we will denote the latter by Λ' .

Let us recall that in globally supersymmetry theory with $n + 1$ chiral multiplets Φ^i , the Yukawa couplings arise from the term

$$\mathcal{L} \supset W_{ij}(\phi) \chi^i \chi^j + \text{h.c.}, \quad i, j = 0, 1, \dots, n, \quad (36)$$

where ϕ^i, χ^i are the scalars and fermions of the chirals and $W_{ij} = \partial^2 W / \partial \phi^i \partial \phi^j$. The potential is

$$V = W_i W^i, \quad (37)$$

where the notation $W^i = (W_i)^\dagger$ is used and let us assume for the moment that the Kähler metric is flat. The values of the fields in the ground state are $\langle \phi^i \rangle = a^i$, $\langle F^i \rangle = f^i$, $\langle \psi_i \rangle = 0$ and the equation of motions give

$$\bar{f}_i = -w_i, \quad w_{ij} f^j = 0, \quad (38)$$

where

$$w_i = W_i(a^i), \quad w_{ij} = W_{ij}(a^i), \quad \dots \quad (39)$$

The term (36) gives then rise to the interaction

$$\mathcal{L} \supset w_{ijk} \delta \phi^k \chi^i \chi^j + \text{h.c.}, \quad (40)$$

where $\delta \phi^i = \phi^i - a^i$. Since supersymmetry is broken, the fermionic shifts will not vanish in the vacuum

$$\langle \delta \chi_i \rangle = -f_i \epsilon. \quad (41)$$

By an appropriate rotation of χ_i , we can define new fermionic fields $\tilde{\chi}_i$

$$\tilde{\chi}_i = R_i^j \chi_j, \quad (42)$$

where R_i^j is an appropriate matrix such that the non-zero fermionic shift are along a specific direction, which we will call it (“0”)

$$\langle \delta \tilde{\chi}_0 \rangle = -f \epsilon, \quad \langle \delta \tilde{\chi}_a \rangle = 0, \quad a = 1, \dots, n, \quad (43)$$

with $|f|^2 = f_i f^i$. Clearly $\tilde{\chi}_0$ is the goldstino, which is defined then as

$$\tilde{\chi}_0 = R_0^i \delta \chi_i \quad (44)$$

and the rest of the modes are given by

$$\delta \tilde{\chi}_a = R_a^i \delta \chi_i. \quad (45)$$

The matrix R_{ij} is orthogonal and chosen to satisfy

$$R_a^i f_i = 0. \quad (46)$$

When this equation is satisfied, then $R_0^i = f_i / |f|$ so that the goldstino is

$$\delta \tilde{\chi}_0 = \frac{f_i}{|f|} \delta \chi_i. \quad (47)$$

Note that instead of rotating χ_i 's, we could have rotated the original superfields Φ^i so that the goldstino belongs to the $\tilde{\Phi}^0$ goldstino superfield, which is a linear combination of the original fields. According to (47), $\tilde{\Phi}^0$ is

$$\tilde{\Phi}_0 = \frac{f_i}{|f|} \Phi^i. \quad (48)$$

The rest of the superfields are given by

$$\tilde{\Phi}_a = R_a^i \Phi^i; \quad (49)$$

therefore, the goldstino is

$$\phi^0 = \frac{f_i}{|f|} \phi^i. \quad (50)$$

The interaction (40) is written then in terms of the new fields as

$$\mathcal{L} \supset R^i_n R^j_m R^k_l w_{ijk} \delta \tilde{\phi}^n \tilde{\chi}^m \tilde{\chi}^l. \quad (51)$$

The possible Yukawa coupling of the goldstino are

$$\mathcal{L}_1 \supset R^i{}_0 R^j{}_0 R^k{}_0 w_{ijk} \delta \tilde{\phi}^0 \tilde{\chi}^0 \tilde{\chi}^0 = |f|^{-3} f^i f^j f^k w_{ijk} \delta \tilde{\phi}^0 \tilde{\chi}^0 \tilde{\chi}^0 = |f|^{-3} s \delta \tilde{\phi}^0 \tilde{\chi}^0 \tilde{\chi}^0, \quad (52)$$

$$\begin{aligned} \mathcal{L}_2 \supset R^i{}_a R^j{}_0 R^k{}_0 w_{ijk} \delta \tilde{\phi}^a \tilde{\chi}^0 \tilde{\chi}^0 &= |f|^{-2} R^i{}_a f^j f^k w_{ijk} \delta \tilde{\phi}^a \tilde{\chi}^0 \tilde{\chi}^0 \\ &= |f|^{-2} R^i{}_a s_i \delta \tilde{\phi}^a \tilde{\chi}^0 \tilde{\chi}^0, \end{aligned} \quad (53)$$

$$\mathcal{L}_2 \supset R^i{}_a R^j{}_b R^k{}_0 w_{ijk} \delta \tilde{\phi}^a \tilde{\chi}^b \tilde{\chi}^0, \quad (54)$$

where

$$s = f^i f^j f^k w_{ijk}, \quad s_k = f^i f^j w_{ijk}. \quad (55)$$

We will show now that

$$s = 0, \quad s_i = 0 \quad (56)$$

so that a globally supersymmetric theory the only trilinear Yukawa coupling is the one that contains only one goldstino or one sgoldstino. For this, we need to recall that the fermionic mass matrix $m_F = w_{ij}$ has a zero eigenvalue

$$m_{Fij} f^j = 0, \quad (57)$$

and the bosonic mass matrix

$$M_B^2 = \begin{pmatrix} m_F^\dagger m_F & \sigma \\ \sigma^\dagger & m_F m_F^\dagger \end{pmatrix}, \quad \sigma_{ij} = w_{ijk} f^k \quad (58)$$

is positive definite

$$\langle \Psi | M_B^2 | \Psi \rangle \geq 0. \quad (59)$$

For

$$|\Psi\rangle = \begin{pmatrix} f_i \\ f^i \end{pmatrix} \quad (60)$$

we get, since m_F annihilates f^i ,

$$\text{Re}(f^i f^j s_{ij}) \geq 0. \quad (61)$$

Moreover, since m_F annihilates also $e^{i\varphi} f^i$, where ϕ is an arbitrary phase, we get in general

$$\text{Re}(e^{2i\varphi} f^i f^j s_{ij}) \geq 0 \quad (62)$$

which leads to

$$s = f^i f^j s_{ij} = f^i f^j f^k w_{ijk} = 0. \quad (63)$$

Therefore, the coupling \mathcal{L}_1 vanishes and there is no (goldstino² sgoldstino) coupling.

We can also prove that there is no (goldstino² scalar) Yukawa coupling by showing that $s_i = 0$, which means that \mathcal{L}_2 vanishes as well. By using (63), it is easy to see that in fact

$$\langle \Psi | M_B^2 | \Psi \rangle = 0 \quad (64)$$

and since M_B^2 is positive definite, M_B^2 annihilates $|\Psi\rangle$

$$M_B^2 |\Psi\rangle = 0. \quad (65)$$

Then, by using (57), (63), we find

$$\sigma_{ij} f^j = w_{ijk} f^j f^k = 0. \quad (66)$$

Therefore, $s_i = 0$ and the interaction \mathcal{L}_2 similarly vanish. As a result, in a globally supersymmetric theory, the only Yukawa coupling that is allowed, is only \mathcal{L}_3 , i.e., a single goldstino interacting with a scalar and a fermion of the matter scalar multiplet or a single sgoldstino interacting with two fermions of the matter scalar multiplet. In particular, this means that there is no way to break supersymmetry just with a single chiral multiplet.

Let us now turn to the general case of a non-flat Kähler metric $g_{i\bar{j}}$. In this case, the bosonic mass matrix is

$$M_B^2 = \begin{pmatrix} -K^j{}_i + (m_F^\dagger m_F)^j{}_i & \sigma \\ \sigma^\dagger & -K_i{}^j + (m_F^\dagger m_F)_i{}^j \end{pmatrix} \quad (67)$$

where

$$K^j{}_i = K_{\bar{j}\bar{i}} = K_{\bar{j}i\bar{m}n} \bar{f}^{\bar{m}} f^k \quad (68)$$

and $K_{\bar{j}i\bar{m}n} = R_{\bar{j}i\bar{m}n}$ in normal coordinates. Now, the corresponding relation (59) for the positivity of M_B^2 does not lead to any conclusive relation. The Yukawa couplings originate from the term

$$\mathcal{L} \supset (W_{ij} - \Gamma_{ij}^k W_k) \chi^i \chi^j + \text{h.c.} \quad (69)$$

which gives rise to

$$\mathcal{L} \supset (W_{ijk} - \partial_k \Gamma_{ij}^l W_l - \Gamma_{ij}^l W_{lk}) \delta\phi^k \chi^i \chi^j + \text{h.c.} \quad (70)$$

Rotating the fields such that again the goldstino is in the 0-direction as before, we get the interaction

$$\mathcal{L} \supset \tilde{s} \delta\phi^0 \chi^0 \chi^0 + \text{h.c.} \quad (71)$$

where

$$\tilde{s} = (W_{ijk} - \partial_k \Gamma_{ij}^l W_l - \Gamma_{ij}^l W_{lk}) f^i f^j f^k. \quad (72)$$

Clearly now $\tilde{s} \neq 0$ as can easily be checked for the simplest case of a linear superpotential $W = f\Phi$. In fact it is easy to see that if the scale of the Kähler manifold is M then the sgoldstino mass is

$$m_s \sim \frac{f}{M} \quad (73)$$

and \tilde{s} is of the order of

$$\tilde{s} \sim \frac{f}{M^2} \sim \frac{m_s^2}{f}. \quad (74)$$

Therefore, the effective coupling in the IR will be schematically of the form

$$\frac{m_s^2}{f} \chi^0 \chi^0 \phi^0 - \frac{1}{2} m_s^2 \phi_0^2 + \dots + \text{h.c.} \quad (75)$$

which gives rise to a term of the form

$$\mathcal{L} \supset \frac{m_s^2}{f^2} (\chi^0 \bar{\chi}^0)^2 \quad (76)$$

after integrating out the sgoldstino. Such a term is suppressed by the scale $\Lambda' = f/m_s$ and therefore it can be ignored as long as it is much larger than the Volkov–Akulov scale \sqrt{f} ($\Lambda' \gg \sqrt{f}$). In this case, interactions like (76) can safely be ignored and the theory will be described by Volkov–Akulov for

$$\frac{f}{m_s} \gg \sqrt{f}. \quad (77)$$

In other words, the Volkov–Akulov description is valid for

$$m_s \ll \sqrt{f} \ll \Lambda'. \quad (78)$$

This limit is the one considered in the models with constraint superfields in which the sgoldstino can be safely integrated out resulting in a non-linearly realized supersymmetric Volkov–Akulov theory for the goldstino mode. The VA description is then valid only up to a UV cutoff equal to the mass m_{lightest} of the lightest particle mixing with the goldstino. This particle can be the sgoldstino or one of the fermions orthogonal to the goldstino. Of course, as in all effective Lagrangians, the VA scale f must obey $f > m_{\text{lightest}}^2$.

4. The complex linear multiplet

We have explicitly demonstrated in the previous section that higher dimensional operators contribute to the vacuum structure and may lead to supersymmetry breaking.

Here we will see that it is possible to break supersymmetry without introducing any Lorentz non-invariant vev.

The reason that the potential (6) cannot break supersymmetry is that the two terms in (6), coming from the two- and four-derivative terms of (5) have the same sign. Clearly, new extrema can emerge only if these terms have opposite sign, i.e. if the first contribution coming from the leading term in (5) flips sign. This can happen for the complex linear multiplet [21,22].

The complex linear or nonminimal multiplet is defined as

$$\bar{D}^2 \Sigma = 0. \quad (79)$$

The constraint (79) above is just the field equation for a free chiral multiplet. Note that if the further constraint $\Sigma = \bar{\Sigma}$ is imposed, the complex linear multiplet turns into a linear one. The standard kinetic Lagrangian for the complex linear superfield in superspace reads

$$\mathcal{L}_0 = - \int d^4\theta \Sigma \bar{\Sigma}. \quad (80)$$

Note the relative minus sign compared to the kinetic Lagrangian of a chiral multiplet. This is necessary for the theory to contain no ghosts. The relative minus sign of the complex linear multiplet Σ compared to the standard kinetic term for a chiral multiplet Φ can be understood in terms of a duality transformation. Indeed, consider the action

$$\mathcal{L}_D = - \int d^4\theta (\Sigma \bar{\Sigma} + \Phi \Sigma + \bar{\Phi} \bar{\Sigma}), \quad (81)$$

where Φ is chiral and Σ is unconstrained. Integrating out Φ we get both Eq. (80) and the constraint (79). However, by integrating out Σ , we get $\Sigma = -\bar{\Phi}$. Plugging back this equality into (81), we get the standard kinetic term of a chiral multiplet

$$\mathcal{L}_0 = \int d^4\theta \Phi \bar{\Phi}. \quad (82)$$

As announced, the overall sign in Lagrangian (82) is opposite to that of (80).

To find the superspace equation of motion, we should express Σ in terms of an unconstrained superfield. This can be done by introducing a general spinor superfield Ψ^α with gauge transformation

$$\delta\Psi_\alpha = D^\beta \Lambda_{(\alpha\beta)} \quad (83)$$

where $\Lambda_{(\alpha\beta)}$ is arbitrary. It is easy to see that by defining

$$\Sigma = \bar{D}_{\dot{\alpha}} \bar{\Psi}^{\dot{\alpha}}, \quad (84)$$

Σ satisfies the constraint (79). Then the field equation following from Eq. (80) is

$$D_\alpha \Sigma = 0. \quad (85)$$

Therefore, the field equation of a complex linear multiplet is just the constraint of a chiral multiplet and, as noticed above, the constraint on a linear is the field equation of a chiral. This indicated the duality between the two kind of multiplets, at least in the free case. The field content of the complex linear multiplet Σ is revealed via the projection over components as

$$\begin{aligned} A &= \Sigma|, \\ \psi_\alpha &= \frac{1}{\sqrt{2}} D_\alpha \bar{\Sigma}|, \\ F &= -\frac{1}{4} D^2 \Sigma|, \\ \lambda_\alpha &= \frac{1}{\sqrt{2}} D_\alpha \Sigma|, \\ P_{\alpha\dot{\beta}} &= \bar{D}_{\dot{\beta}} D_\alpha \Sigma|, \quad \bar{P}_{\alpha\dot{\beta}} = -D_\beta \bar{D}_{\dot{\alpha}} \bar{\Sigma}|, \\ \chi_\alpha &= \frac{1}{2} \bar{D}_{\dot{\alpha}} D_\alpha \bar{D}^{\dot{\alpha}} \bar{\Sigma}|, \quad \bar{\chi}_{\dot{\alpha}} = \frac{1}{2} D^\alpha \bar{D}_{\dot{\alpha}} D_\alpha \Sigma|. \end{aligned} \quad (86)$$

In other words, a complex linear multiplet contains a chiral multiplet (A, λ_α, F) and an antichiral spinor superfield $(\psi_\alpha, P_{\alpha\dot{\beta}}, \chi_\alpha)$. Therefore, the complex linear multiplet is a reducible $12 + 12$ dimensional representation of the $\mathcal{N} = 1$ supersymmetry. It should be noted that since Σ is not chiral, there is no superpotential and there are no supersymmetric non-derivative interactions. However, the complex linear multiplet can still be consistently coupled to ordinary vector multiplets of the $\mathcal{N} = 1$ theory.

We give for later use the supersymmetry transformations of the fermionic components of Σ

$$\delta\psi_\alpha = \sqrt{2} i \sigma_{\alpha\dot{\beta}}^\mu \bar{\xi}^{\dot{\beta}} \partial_\mu \bar{A} - \frac{1}{\sqrt{2}} \bar{\xi}^{\dot{\beta}} \bar{P}_{\alpha\dot{\beta}}, \quad (87)$$

$$\delta\chi_\alpha = 2i \sigma_{\alpha\dot{\alpha}}^\nu \bar{\sigma}^{\mu\dot{\alpha}\beta} \xi_\beta \partial_\mu \bar{P}_\nu + i \sigma_{\alpha\dot{\alpha}}^\mu \bar{\sigma}^{\nu\dot{\alpha}\beta} \xi_\beta \partial_\mu \bar{P}_\nu - 4\xi_\alpha \partial^2 \bar{A} + 2i \sigma_{\alpha\dot{\alpha}}^\mu \bar{\xi}^{\dot{\alpha}} \partial_\mu \bar{F}, \quad (88)$$

$$\delta\lambda_\alpha = \sqrt{2} \xi_\alpha F - \frac{1}{\sqrt{2}} \bar{\xi}^{\dot{\beta}} P_{\alpha\dot{\beta}}. \quad (89)$$

The transformation rules of the bosonic sector of the complex linear multiplet are

$$\delta A = \sqrt{2}\bar{\xi}\bar{\psi} + \sqrt{2}\xi\lambda, \quad (90)$$

$$\delta F = \frac{i}{\sqrt{2}}\bar{\xi}\bar{\sigma}^\mu\partial_\mu\lambda + \frac{1}{2}\bar{\xi}\bar{\chi}, \quad (91)$$

$$\delta P_{\alpha\dot{\beta}} = -2\sqrt{2}i\xi^\gamma\sigma_{\gamma\dot{\beta}}^\mu\partial_\mu\lambda_\alpha + \sqrt{2}i\xi_\alpha\sigma_{\dot{\beta}}^\mu\partial_\mu\lambda_\beta - \xi_\alpha\bar{\lambda}_{\dot{\beta}} - 2\sqrt{2}i\bar{\xi}_{\dot{\beta}}\sigma_{\alpha\dot{\rho}}^\mu\partial_\mu\bar{\psi}^{\dot{\rho}}. \quad (92)$$

In terms of the components of Σ , Lagrangian (80) is explicitly written as

$$\mathcal{L}_0 = A\partial^2\bar{A} - F\bar{F} + i\partial_\mu\bar{\psi}\bar{\sigma}^\mu\psi + \frac{1}{2}P_\mu\bar{P}^\mu + \frac{1}{2\sqrt{2}}(\chi\lambda + \bar{\chi}\bar{\lambda}). \quad (93)$$

The complex vector P_μ , the complex scalar F and the spinors λ , χ are auxiliary fields. Note that the minus sign in front of the superspace action (80) guarantees that the scalar A is a normal field and not a ghost. However, this choice of sign has flipped the sign of the $F\bar{F}$ relative to the action for a chiral multiplet. This flip of sign is of fundamental importance for what follows and leads to supersymmetry breaking.

5. SUSY breaking by complex linear multiplets

As we have noticed before, although one can couple the linear multiplet to gauge fields [23–27], one cannot write down mass terms or non-derivative interactions as in the chiral multiplet case by means of a superpotential. So, the best we can hope for is to introduce a potential indirectly by using the higher dimensional operators first discussed in [1]. The idea of [1] has been recently revisited and the emergent potential for chiral and vector multiplets has been discussed in [17–19].

To achieve this, we introduce the following Lagrangian in superspace

$$\mathcal{L}_{EP} = \int d^4\theta \frac{1}{64f^2} D^\alpha \Sigma D_\alpha \Sigma \bar{D}_{\dot{\alpha}} \bar{\Sigma} \bar{D}^{\dot{\alpha}} \bar{\Sigma}, \quad (94)$$

where \sqrt{f} is a mass scale. Then, the theory is described by

$$\begin{aligned} \mathcal{L}_\Sigma &= \mathcal{L}_0 + \mathcal{L}_{EP} \\ &= \int d^4\theta \left(-\Sigma \bar{\Sigma} + \frac{1}{64f^2} D^\alpha \Sigma D_\alpha \Sigma \bar{D}_{\dot{\alpha}} \bar{\Sigma} \bar{D}^{\dot{\alpha}} \bar{\Sigma} \right). \end{aligned} \quad (95)$$

By using the unconstrained superfield Φ_α , we find that the field equations are

$$D_\alpha \Sigma + \frac{1}{32f^2} D_\alpha \bar{D}_{\dot{\alpha}} (D^\beta \Sigma D_\beta \Sigma \bar{D}^{\dot{\alpha}} \bar{\Sigma}) = 0. \quad (96)$$

Clearly, the above equation always admits the supersymmetry preserving solution

$$D_\alpha \Sigma = 0. \quad (97)$$

We are interested to investigate if another, supersymmetry breaking solution to (96) exists.

The component form of the bosonic part of Eq. (94) is

$$\mathcal{L}_{EP}^B = \frac{1}{64f^2} (P^\mu P_\mu \bar{P}^\nu \bar{P}_\nu + 4P_\mu \bar{P}^\mu F \bar{F} + 16F^2 \bar{F}^2), \quad (98)$$

so that the bosonic part of the full Lagrangian (95) turns out to be

$$\begin{aligned}\mathcal{L}^B = & -F\bar{F} + A\partial^2\bar{A} + \frac{1}{2}P_\mu\bar{P}^\mu \\ & + \frac{1}{64f^2}(P^\mu P_\mu\bar{P}^\nu\bar{P}_\nu + 4P_\mu\bar{P}^\mu F\bar{F} + 16F^2\bar{F}^2).\end{aligned}\quad (99)$$

From the equations of motion for the complex auxiliary vector we find that

$$P_\mu = 0, \quad (100)$$

whereas the equations of motion for the auxiliary scalar turns out to be

$$F\left(1 - \frac{1}{2f^2}F\bar{F}\right) = 0. \quad (101)$$

There are now two solutions:

$$(i) \quad F = 0, \quad (102)$$

$$(ii) \quad F\bar{F} = 2f^2. \quad (103)$$

Clearly, as it follows from Eqs. (87), (88), (89), the first vacuum $F = 0$ is the supersymmetric one, where supersymmetry is exact. However, the second vacuum, described by the solution (103), explicitly breaks supersymmetry. We note that the theories with $F = 0$ and $F \neq 0$ should not be thought as phases of the same theory but rather as two different theories. This can be illustrated by the following example. Consider a scalar A and an auxiliary field Y with Lagrangian:

$$\mathcal{L}_{AY} = -\frac{1}{2}\partial_\mu A\partial^\mu A - \frac{1}{2}Y^2(aA^2 + b) + \frac{1}{4}Y^4. \quad (104)$$

Solving for Y we get two solutions: $Y = 0$, which gives the free scalar Lagrangian

$$\mathcal{L}_A = -\frac{1}{2}\partial_\mu A\partial^\mu A, \quad (105)$$

and

$$Y^2 = aA^2 + b, \quad (106)$$

which gives the interacting Lagrangian

$$\mathcal{L}'_A = -\frac{1}{2}\partial_\mu A\partial^\mu A - \frac{1}{4}(aA^2 + b)^2. \quad (107)$$

No transition either perturbative or nonperturbative can occur between the two, precisely because the equations for Y are algebraic, so they are truly two different theories.

It should also be noted that the susy-breaking vacuum is specified by the modulus of the auxiliary field F . So, F itself is specified only up to a phase. This is expected due to the invariance of Lagrangian (95) under the global $U(1)$ transformation

$$\Sigma \rightarrow e^{i\phi}\Sigma. \quad (108)$$

For completeness, we give the component form of Lagrangian (95)

$$\begin{aligned}\mathcal{L}_\Sigma = & A\partial^2\bar{A} - F\bar{F} + i\partial_\mu\bar{\psi}\bar{\sigma}^\mu\psi + \frac{1}{2}P_\mu\bar{P}^\mu + \frac{1}{2\sqrt{2}}(\chi\lambda + \bar{\chi}\bar{\lambda}) \\ & + \frac{1}{64f^2}\left\{4(\lambda^\alpha\partial^2\lambda_\alpha)\bar{\lambda}^2 + 2\sqrt{2}i(\partial_\mu\bar{\chi}\bar{\sigma}^\mu\lambda)\bar{\lambda}^2 - 16F\partial^2A\bar{\lambda}^2 + 8iF\partial^\mu P_\mu\bar{\lambda}^2\right.\end{aligned}$$

$$\begin{aligned}
& + 8\partial^2 A \bar{\lambda} \bar{\sigma}^\kappa \lambda \bar{P}_\kappa + 4i \bar{\lambda} \bar{\sigma}^\kappa \sigma^\nu \bar{\sigma}^\mu \lambda \bar{P}_\kappa \partial_\mu P_\nu \\
& + 8i \bar{\lambda} \bar{\sigma}^\kappa \sigma^\mu \partial_\mu \bar{\psi} F \bar{P}_\kappa - 16 \partial_\mu \bar{\psi} \bar{\sigma}^\mu \lambda \partial_\nu \psi \sigma^\nu \bar{\lambda} + 4i \partial_\mu \bar{\psi} \bar{\sigma}^\mu \lambda \bar{P}^2 \\
& + \frac{1}{2} \Omega^{\beta\dot{\beta}\alpha} \Omega_{\beta\dot{\beta}\alpha} \bar{\lambda}^2 - 8i \bar{\lambda}^2 P^\kappa \partial_\kappa F \\
& + \sqrt{2} \bar{P}_\mu \bar{\lambda} \bar{\sigma}^{\mu\dot{\alpha}\beta} \Omega_{\beta\dot{\beta}\alpha} \bar{\sigma}^{\kappa\dot{\beta}\alpha} P_\kappa + 4i P^2 \partial_\mu \psi \sigma^\mu \bar{\lambda} + P^2 \bar{P}^2 \\
& - 8\sqrt{2} F \bar{\chi} \bar{\lambda} \bar{F} - 8 F \bar{F} P_\nu \bar{P}^\nu - 2\sqrt{2} \chi \sigma^\mu \bar{\lambda} P_\mu F \\
& + 4i F P_\mu \bar{\lambda} \bar{\sigma}^\mu \sigma^\nu \partial_\nu \bar{\lambda} - 16i \lambda \sigma^\nu \bar{\lambda} \bar{F} \partial_\nu F \\
& + 2\sqrt{2} \bar{P}_\nu \bar{\sigma}^{\nu\dot{\beta}\beta} \Omega_{\beta\dot{\beta}\alpha} \chi^\alpha \bar{F} - 2\Omega_{\beta\dot{\beta}\alpha} \chi^\beta \bar{\lambda} \bar{\sigma}^\beta \lambda^\alpha + 2\sqrt{2} i \partial_\mu \bar{\lambda} \bar{\sigma}^{\mu\dot{\rho}\beta} \Omega_{\beta\dot{\beta}\alpha} \lambda^\alpha \bar{\lambda}^{\dot{\beta}} \\
& - 8i \partial_\nu \psi \sigma^\nu \bar{\sigma}^\mu \lambda P_\mu \bar{F} - \sqrt{2} \lambda \sigma^\mu \bar{\sigma}^\nu \chi P_\mu \bar{P}_\nu - 2i \lambda \sigma^\kappa \bar{\sigma}^\mu \sigma^\nu \partial_\nu \bar{\lambda} P_\kappa \bar{P}_\mu \\
& - 8 \lambda \sigma^\nu \bar{\lambda} P_\nu \partial^2 \bar{A} - 8i \lambda \sigma^\nu \bar{\lambda} P_\nu \partial_\mu \bar{P}^\mu \\
& + 16 F^2 \bar{F}^2 - 8\sqrt{2} \lambda \chi F \bar{F} - 16i \lambda \sigma^\nu \partial_\nu \bar{\lambda} F \bar{F} \\
& - 16 \lambda^2 \bar{F} \partial^2 \bar{A} - 16i \lambda^2 \bar{F} \partial_\mu \bar{P}^\mu - \lambda^2 \Xi^2 \Big\}, \tag{109}
\end{aligned}$$

where

$$\begin{aligned}
\Omega^{\beta\dot{\beta}\alpha} &= -2\sqrt{2} i \bar{\sigma}^{\mu\dot{\beta}\beta} \partial_\mu \lambda^\alpha - i \sqrt{2} \epsilon^{\beta\alpha} \bar{\sigma}^{\mu\dot{\beta}\gamma} \partial_\mu \lambda_\gamma - \epsilon^{\beta\alpha} \bar{\chi}^{\dot{\beta}}, \\
\Omega_{\rho\dot{\rho}\sigma} &= \epsilon_{\rho\beta} \epsilon_{\rho\dot{\beta}} \epsilon_{\sigma\alpha} \Omega^{\beta\dot{\beta}\alpha} \tag{110}
\end{aligned}$$

and

$$\Xi_\beta = \chi_\beta + \sqrt{2} i \sigma_{\beta\dot{\beta}}^\nu \partial_\nu \bar{\lambda}^{\dot{\beta}}. \tag{111}$$

We should note that Lagrangian (109) contains also first derivatives of the auxiliaries F , P_μ , χ . Therefore, one may question if these fields are really auxiliaries. However, it can easily be checked that these derivative terms are always multiplied by fermions. Therefore their equations of motion can be integrated by iteration in a power series of the fermions, which terminates due to the nilpotent nature of the latter.

To identify the goldstino mode, one should look at the supersymmetry transformations and, in particular, to the fermion shifts. It is then easy to recognize that since

$$\delta \lambda_\alpha = 2\xi_\alpha f + \dots, \tag{112}$$

the goldstino of the broken supersymmetry is proportional to λ , i.e., one of the auxiliary fermions. Here something unusual has happened; namely, an auxiliary fermion has turned into a goldstino mode in the susy breaking vacuum. However, the latter is propagating and the vacuum (103) should definitely give rise to a kinetic term for λ . Indeed, it is straightforward to see that the higher dimensional operator Lagrangian gives rise to the following coupling for the auxiliary fermion λ

$$\mathcal{L}_{EP} \supset \left(\frac{1}{4f^2} F \bar{F} \right) i \partial_\mu \bar{\lambda} \bar{\sigma}^\mu \lambda. \tag{113}$$

In the susy breaking vacuum obtained from Eq. (101) we have

$$\langle F \bar{F} \rangle = 2f^2, \tag{114}$$

leading to a standard fermionic kinetic term with the correct sign

$$\mathcal{L}_{EP} \supset \frac{i}{2} \partial_\mu \bar{\lambda} \bar{\sigma}^\mu \lambda. \quad (115)$$

Therefore, on the susy breaking vacuum (103), the auxiliary fermion λ is propagating and it is proportional to the goldstino mode of broken susy. Note that due to the model independent relation (114), the kinetic term (115) for the goldstino is also model independent. In fact what has happened here is that the susy breaking phase is a realization of non-linear supersymmetry.

We should also mention that the fermion bilinears $\chi\lambda$ and $\bar{\chi}\bar{\lambda}$ appear in the action as

$$\mathcal{L}_\Sigma \supset \frac{1}{2\sqrt{2}} \left(1 - \frac{F\bar{F}}{2f^2} \right) (\chi\lambda + \bar{\chi}\bar{\lambda}). \quad (116)$$

Such terms vanish on the non-supersymmetric vacuum and protect the theory from unwanted, dangerous terms. Moreover, as in the spinor superfield and real multiplet case, there is no superpartner of the goldstino. In fact, the propagating modes are the real scalar A , the fermion ψ and the goldstino λ , which definitely do not form a multiplet of the (broken) susy. The reason again is that the rest of the fields of the complex linear multiplet are auxiliaries and therefore the goldstino decouples.

One could proceed and solve the field equations for the auxiliaries in (109). Although this is a formidable task, there is an indirect way to proceed in superspace. We will show below that the theory (109) describes a free chiral multiplet and a constraint chiral superfield which describes a Volkov–Akulov mode. To see how this happens, let us remind briefly some aspects of non-linear supersymmetry realizations. It is well known that the following Lagrangian [5]

$$\mathcal{L} = \int d^4\theta X_{NL} \bar{X}_{NL} + \sqrt{2}f \left(\int d^2\theta X_{NL} + \text{h.c.} \right) + \left(\int d^2\theta \Psi X_{NL}^2 + \text{h.c.} \right) \quad (117)$$

is on-shell equivalent to the Akulov–Volkov theory. In fact, the Lagrange multiplier chiral superfield Ψ imposes the constraint

$$X_{NL}^2 = 0 \quad (118)$$

on the chiral superfield X_{NL} , leads to the non-linear realization of supersymmetry [3,5,6] and reproduces the Volkov–Akulov model. The Lagrangian (117) gives rise to the following two equations of motion in superspace

$$-\frac{1}{4} \bar{D}^2 \bar{X}_{NL} + \sqrt{2}f + 2\Psi X_{NL} = 0, \quad (119)$$

$$X_{NL}^2 = 0. \quad (120)$$

The theory we consider here is described by the Lagrangian

$$\mathcal{L} = - \int d^4\theta \Sigma \bar{\Sigma} + \int d^4\theta \frac{1}{64f^2} D^\alpha \Sigma D_\alpha \Sigma \bar{D}_{\dot{\alpha}} \bar{\Sigma} \bar{D}^{\dot{\alpha}} \bar{\Sigma} \quad (121)$$

and the superfield equations of motion are written as

$$D_\alpha \Sigma + \frac{1}{32f^2} D_\alpha \bar{D}_{\dot{\alpha}} (D^\beta \Sigma D_\beta \Sigma \bar{D}^{\dot{\alpha}} \bar{\Sigma}) = 0. \quad (122)$$

These equations can equivalently be expressed as

$$\Sigma = - \frac{1}{32f^2} \bar{D}_{\dot{\alpha}} (D^\beta \Sigma D_\beta \Sigma \bar{D}^{\dot{\alpha}} \bar{\Sigma}) + \bar{\Phi} \quad (123)$$

where Φ is a chiral superfield. Hitting the above equation with \bar{D}^2 leads to a consistency condition

$$\bar{D}^2 \bar{\Phi} = 0, \quad (124)$$

which implies that Φ is a free chiral superfield. In fact, Σ can be written as

$$\Sigma = H + \bar{\Phi}, \quad (125)$$

where H satisfies the equations of motion

$$H = -\frac{1}{32f^2} \bar{D}_{\dot{\alpha}} (D^{\beta} H D_{\beta} H \bar{D}^{\dot{\alpha}} \bar{H}). \quad (126)$$

It is now straightforward to solve Eq. (126) in terms of a constrained chiral superfield subject to (119) and (120) by identifying H (up to a phase) with the goldstino chiral superfield X_{NL}

$$H = X_{NL}. \quad (127)$$

Let us verify that (127) indeed solves (126). From (120) one finds

$$D^{\beta} X_{NL} D_{\beta} X_{NL} = -X_{NL} D^2 X_{NL}, \quad (128)$$

whereas, (119) gives

$$X_{NL} \bar{D}^2 \bar{X}_{NL} = 4\sqrt{2}f X_{NL}, \quad (129)$$

$$X_{NL} D^2 X_{NL} = 4\sqrt{2}f X_{NL} + 8X_{NL} \bar{X}_{NL} \bar{\Psi}. \quad (130)$$

For the right part of (126), by using (127) we have

$$\begin{aligned} & -\frac{1}{32f^2} \bar{D}_{\dot{\alpha}} (D^{\beta} X_{NL} D_{\beta} X_{NL} \bar{D}^{\dot{\alpha}} \bar{X}_{NL}) \\ &= \frac{1}{32f^2} \bar{D}_{\dot{\alpha}} (X_{NL} D^2 X_{NL} \bar{D}^{\dot{\alpha}} \bar{X}_{NL}) \\ &= \frac{1}{32f^2} \bar{D}_{\dot{\alpha}} \{ (4\sqrt{2}f X_{NL} + 8X_{NL} \bar{X}_{NL} \bar{\Psi}) \bar{D}^{\dot{\alpha}} \bar{X}_{NL} \} \\ &= \frac{1}{32f^2} \bar{D}_{\dot{\alpha}} \{ (4\sqrt{2}f X_{NL}) \bar{D}^{\dot{\alpha}} \bar{X}_{NL} \} \\ &= \frac{1}{4\sqrt{2}f} X_{NL} \bar{D}^2 \bar{X}_{NL} \\ &= X_{NL}, \end{aligned}$$

where we have used the identities (120), (128), (129) and (130). Thus, the equations of motion for the superfield Σ are solved in terms of a free chiral multiplet ($D^2 \Phi = 0$), and a constrained chiral superfield ($H = X_{NL}$). Therefore, Σ describes on-shell a free chiral multiplet and a goldstino superfield. We should note however, that although (127) is a solution, we have not proven that it is unique.

The component fields of the Σ multiplet can be deduced from the relation

$$\Sigma = X_{NL} + \bar{\Phi}. \quad (131)$$

From Eq. (131) the fields F and λ_{α} of Σ are identified as the appropriate component fields of the constrained chiral superfield X_{NL} since

$$\lambda_\alpha = \frac{1}{\sqrt{2}} D_\alpha \Sigma \Big| = \frac{1}{\sqrt{2}} D_\alpha X_{NL} \Big| \quad (132)$$

and

$$F = -\frac{1}{4} D^2 \Sigma \Big| = -\frac{1}{4} D^2 X_{NL} \Big|. \quad (133)$$

Thus, we can deduce their equations of motion just from the X_{NL} . On-shell we have

$$X_{NL} = \frac{\lambda^2}{2F} + \sqrt{2}\theta\lambda + \theta^2 F \quad (134)$$

with [3]

$$F = -\sqrt{2}f \left(1 + \frac{\bar{\lambda}^2}{16f^4} \partial^2 \lambda^2 - \frac{3}{256f^8} \lambda^2 \bar{\lambda}^2 \partial^2 \lambda^2 \partial^2 \bar{\lambda}^2 \right), \quad (135)$$

$$i\bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial_\mu \lambda_\alpha = \frac{1}{4f^2} \bar{\lambda}^{\dot{\alpha}} \partial^2 \lambda^2 - \frac{1}{64f^6} \bar{\lambda}^{\dot{\alpha}} \lambda^2 \partial^2 \lambda^2 \partial^2 \bar{\lambda}^2 - \frac{1}{64f^6} \bar{\lambda}^{\dot{\alpha}} \partial^2 (\lambda^2 \bar{\lambda}^2 \partial^2 \lambda^2). \quad (136)$$

Eq. (136) is the equation of motion for the goldstino and Eq. (135) is the solution for F in terms of the goldstino as anticipated. From the chiral multiplet we can easily identify ψ_α as the fermion of the chiral multiplet Φ , since

$$\psi_\alpha = \frac{1}{\sqrt{2}} D_\alpha \bar{\Sigma} \Big| = \frac{1}{\sqrt{2}} D_\alpha \Phi \Big|. \quad (137)$$

On-shell, Φ is a free chiral superfield so that

$$\Phi = A_\Phi + \sqrt{2}\theta\psi + \theta^2 F_\Phi \quad (138)$$

with

$$\partial^2 A_\Phi = 0, \quad (139)$$

$$\bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial_\mu \psi_\alpha = 0, \quad (140)$$

$$F_\Phi = 0. \quad (141)$$

Thus, ψ_α is a free massless fermion. From (131) we have, for the scalar component A of Σ

$$A = \bar{A}_\Phi + \frac{\lambda^2}{2F}, \quad (142)$$

so that this component of Σ is solved in terms of the free scalar of the chiral multiplet and the goldstino. The last two auxiliary fields P_μ and χ_α can be specified similarly. For the complex vector auxiliary P_μ we have

$$P_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}} D_\alpha \Sigma \Big| = \bar{D}_{\dot{\alpha}} D_\alpha X_{NL} \Big| = -2i\sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \left(\frac{\lambda^2}{2F} \right) \quad (143)$$

whereas for χ_α we find

$$\chi_\alpha = \frac{1}{2} \bar{D}_{\dot{\alpha}} D_\alpha \bar{D}^{\dot{\alpha}} \bar{\Sigma} \Big| = \frac{1}{2} \bar{D}_{\dot{\alpha}} D_\alpha \bar{D}^{\dot{\alpha}} \bar{X}_{NL} \Big| = i\sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \bar{\lambda}^{\dot{\alpha}}. \quad (144)$$

Such a model of susy breaking can be considered as a hidden sector. Then, couplings to the visible sector can be introduced through the interactions

$$\mathcal{L}_{\text{int}} = -\frac{m_i^2}{2f^2} \int d^4\theta \Sigma \bar{\Sigma} \Phi^i \bar{\Phi}^i - \frac{m_g}{4f^2} \int d^4\theta \Sigma \bar{\Sigma} (W^\alpha W_\alpha + \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}) \quad (145)$$

where Φ^i are chiral matter in the visible sector and W_α is the supersymmetric field strength of vectors. In the susy breaking vacuum, m_i, m_g are just soft masses for the scalars of the chiral multiplets of the visible sector and the gauginos, respectively.

As a final comment, we should point out that the scale \sqrt{f} is not the cutoff of the theory. Since the theory (109) describes a free chiral multiplet and a constraint chiral superfield (Volkov–Akulov), the discussion of Section 3 applies. In particular, the cutoff is $\Lambda' = f/m_s$ and this difference between \sqrt{f} and Λ' is common to models with constraint superfields like [3,10], which are unique up to superspace derivative terms, due to $X^2 = 0$ constraint.

6. Conclusions

It has been advocated in [1] that the addition of higher dimension operators to a supersymmetric theory may lead to the appearance of new vacua, where only one of them is continuously connected to the standard theory in the limit of removing the higher dimension operators. This is possible, if the equations of motion for the auxiliaries have more than one solutions which satisfy the appropriate conditions. In [1], some examples were discussed, none of which however realized that proposal. Here we have provided an example, where the proposal works. This is achieved by employing a complex linear multiplet, in which the quadratic term of its scalar auxiliary fields has opposite sign of the corresponding term in a chiral multiplet action. Therefore, by adding an appropriate ghost-free higher dimension operator, a potential is induced according to [1,17–19]. This potential, has a second non-supersymmetric vacuum at a non-zero value of the scalar auxiliary besides the supersymmetric one. In the susy breaking vacuum, the propagating fields are the scalar and the fermion of the complex linear multiplet and the goldstino mode of the broken supersymmetry. Interesting enough, the goldstino mode turns out to be one of the auxiliary fermions of the complex linear multiplet, which now propagates in the new non-supersymmetric vacuum. The coupling of this model to supergravity is an interesting project that we leave for future work.

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Appendix A

It should be noted that, instead of (94), one could consider the following more general Lagrangian

$$\mathcal{L}'_{EP} = \int d^4\theta \frac{1}{64} \mathcal{U}(\Sigma, \bar{\Sigma}) D^\alpha \Sigma D_\alpha \Sigma \bar{D}_{\dot{\alpha}} \bar{\Sigma} \bar{D}^{\dot{\alpha}} \bar{\Sigma}, \quad (146)$$

where, \mathcal{U} is a real, strictly positive, but otherwise arbitrary function of Σ and $\bar{\Sigma}$ with mass dimension (-4) . As we will see in the moment, a potential emerges for the complex scalar A of the complex linear multiplet Σ . The component form of the bosonic part of Eq. (146) is

$$\mathcal{L}'_{EP} = \frac{1}{64} \mathcal{U} P^\mu P_\mu \bar{P}^\nu \bar{P}_\nu + \frac{1}{16} P_\mu \bar{P}^\mu \mathcal{U} F \bar{F} + \frac{1}{4} \mathcal{U} F^2 \bar{F}^2, \quad (147)$$

where $\mathcal{U} = \mathcal{U}(A, \bar{A}) = \mathcal{U}(\Sigma, \bar{\Sigma})$. Then, the bosonic part of the Lagrangian

$$\begin{aligned} \mathcal{L}'_\Sigma &= \mathcal{L}_0 + \mathcal{L}'_{EP} \\ &= \int d^4\theta \left(-\Sigma \bar{\Sigma} + \frac{1}{64} \mathcal{U}(\Sigma, \bar{\Sigma}) D^\alpha \Sigma D_\alpha \bar{\Sigma} \bar{D}^{\dot{\alpha}} \bar{\Sigma} \bar{D}_{\dot{\alpha}} \bar{\Sigma} \right), \end{aligned} \quad (148)$$

is

$$\begin{aligned} \mathcal{L}'^B &= -F \bar{F} + A \partial^2 \bar{A} - \frac{1}{2} P_\mu \bar{P}^\mu \\ &\quad + \frac{1}{64} \mathcal{U} P^\mu P_\mu \bar{P}^\nu \bar{P}_\nu + \frac{1}{16} P_\mu \bar{P}^\mu \mathcal{U} F \bar{F} + \frac{1}{4} \mathcal{U} F^2 \bar{F}^2. \end{aligned} \quad (149)$$

From the equations of motion for the complex auxiliary vector we find again that

$$P_\mu = 0, \quad (150)$$

whereas the equations of motion for the auxiliary scalar are now

$$F \left(1 - \frac{\mathcal{U}}{2} F \bar{F} \right) = 0. \quad (151)$$

There are again two solutions:

$$(i) \quad F = 0, \quad (152)$$

$$(ii) \quad F \bar{F} = \frac{2}{\mathcal{U}(A, \bar{A})}. \quad (153)$$

The first is the supersymmetric one while the second breaks supersymmetry. Plugging back Eqs. (150) and (151) into (149) we find

$$\mathcal{L}^B = A \partial^2 \bar{A} - \frac{1}{\mathcal{U}(A, \bar{A})}. \quad (154)$$

We see now that a potential has emerged

$$\mathcal{V}_{EP} = \frac{1}{\mathcal{U}(A, \bar{A})}. \quad (155)$$

For example one can have

$$\mathcal{U}(A, \bar{A}) = \frac{1}{f^2 + m_A^2 A \bar{A}} \quad (156)$$

where f is a mass scale. This case leads to a scalar potential

$$\mathcal{V} = f^2 + m_A^2 A \bar{A} \quad (157)$$

i.e. to a mass for the scalar A . The minimum of potential (157) is at $A = 0$, which is a supersymmetry breaking vacuum since

$$\langle F\bar{F} \rangle = 2f^2 \neq 0. \quad (158)$$

Another example is provided by

$$\mathcal{U}(A, \bar{A}) = \frac{1}{f^2 + \frac{\lambda}{4!}(A\bar{A} - \mu^2)^2}, \quad (159)$$

which gives rise to a potential

$$V = f^2 + \frac{\lambda}{4!}(A\bar{A} - \mu^2)^2. \quad (160)$$

In this case, the $U(1)$ global symmetry $A \rightarrow e^{i\alpha} A$ is broken at the vacuum $A\bar{A} = \mu^2$ where susy is also broken because

$$\langle F\bar{F} \rangle = 2f^2 \neq 0. \quad (161)$$

In general, the complex scalar multiplet can have an arbitrary potential in the susy breaking vacuum, specified by the arbitrary real positive function $\mathcal{U}(A, \bar{A})$.

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